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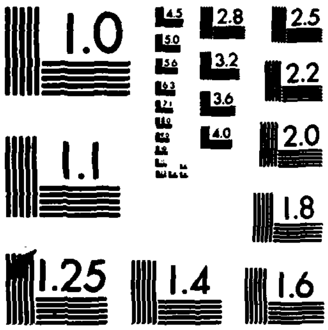

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Popular failure criteria of fiber-reinforced composite materials are described and compared. These criteria are empirical and should only be judged from the standpoint of the fitness to data and the ease of application. The criteria for orthotropic plies of unidirectional composites are extensions of those for isotropic materials. The quadratic criteria are considered to be the most suitable for both isotropic and composite materials. Macroscopic criteria are essential for design and for providing guidelines for (OVER)

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materials improvements. Strictly speaking, failure criteria for multidirectional laminants are valid up to the first-ply failure envelope; i.e., before transverse cracking and delamination occur. Finally, conditions for fully optimized ply properties are easily derived from the quadratic failure criterion.

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A Survey of Macroscopic Failure Criteria for Composite Materials*

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Abstract - Popular failure criteria of fiber-reinforced composite materials are described and compared. These criteria are empirical and should only be judged from the standpoint of the fitness to data and the ease of application. The criteria for orthotropic plies of unidirectional composites are extensions of those for isotropic materials. The quadratic criteria are considered to be the most suitable for both isotropic and composite materials. Macroscopic criteria are essential for design and for providing guidelines for materials improvements. Strictly speaking, failure criteria for multidirectional laminants are valid up to the first-ply failure envelope; i.e., before transverse cracking and delamination occur. Finally, conditions for fully optimized ply properties are easily derived from the quadratic failure criterion.

1. Introduction

Failure criteria have been in use for structural materials for centuries. They can be divided into non-interactive criteria, such as the maximum stress or maximum strain; and interactive criteria, such as the quadratic approximation. For brittle materials, the non-interactive criteria are commonly used; for ductile materials with yielding, the interactive criteria are used. For fiber-reinforced composite materials the popular failure criteria are, as expected, extensions of those for isotropic materials. Material symmetry and tensor transformation properties necessarily impose certain restrictions on

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the algebraic form or the geometric shape of the failure criteria. Such restrictions are present for both isotropic and orthotropic materials. It is therefore useful to first examine these restrictions as they are applied to isotropic materials. A comparison of similar criteria for orthotropic materials in terms of flexibility, usefulness, and implications in materials improvements will be presented. We will limit our discussion to the plane stress criteria as they appear in stress and strain space.

2. Maximum Stress Criteria for Isotropic Materials

For isotropic materials, the non-interactive failure criteria can be derived from an idealized uniaxial tension test shown in Figure 1. The maximum stress criterion for the normal stresses is:

$$|\sigma_x| < X \qquad |\sigma_y| < X \tag{1}$$

Graphically, (1) is shown in Figure 2. If the uniaxial compressive strength X' is different from tensile, the maximum stress criterion is

$$\sigma_x \leq X,\ \sigma_y \leq X \qquad |-\sigma_x| \leq X',\ |-\sigma_y| \leq X' \tag{2}$$

Graphically, (2) is shown in Figure 3.

Since stress components are governed by their transformation equations, failure of an isotropic material must be independent of the rotation of the coordinate axes. Thus, the failure of combined tension and compression stresses must be equal to that of a pure shear. This is shown in Figure 4 using the Mohr's circle. In fact failure is the same for any combined stress state on the same Mohr's circle; i.e., failure will occur when I and R reach certain prescribed values independent of the phase angle θ_o. Invariants I,

R, and phase angle θ_o are defined

$$\begin{aligned} I = p &= 1/2\ (\sigma_x + \sigma_y) \\ R^2 = q^2 + r^2 &= 1/4\ (\sigma_x - \sigma_y)^2 + \sigma_s^2 \\ 2\theta_o &= \tan^{-1}\ (q/r) \end{aligned} \tag{3}$$

Because of the restriction imposed by the stress transformation equations, the maximum stress criterion shown in Figure 2 must be modified by truncating the corners in the second and fourth quadrants. This is done in Figure 5, where the p, q, and r axes are defined in (3) above. The lines that truncate the failure envelope represent constant maximum shear strengths. This is commonly known as the Tresca criterion. Based on this criterion,

$$S = X/2. \tag{4}$$

If the full failure envelope is drawn in stress-space, the cross section in any constant p plane or in any q-r plane (where r is the third dimension) must be circular. This stress space is also the Mohr's circle space with the circle drawn normal to the p-axis in Figure 5. The combined restrictions imposed by isotropy and stress transformation lead to a cylindrical failure envelope for the maximum stress criterion; the generator is the p-axis in Figure 5. The failure envelope is a circular cylinder with conical heads.

3. Maximum Strain Criteria for Isotropic Materials

Analogous to the maximum stress criterion, the biaxial failure strains due to uniaxial tension and compression tests are shown in Figure 6 where tensile and compressive strengths are assumed to be equal, and Poisson's ratio, to be 1/3. The failure envelope in strain space is also constrained by isotropy and strain transformation equations similar to that in the stress space. An admissible strain criterion is shown in Figure 7. Note the constant maximum shear strain lines are drawn in the second and fourth quadrants.

To be consistent with the relation between the tensile and shear strengths postulated in (4); i.e.,

$$S = X/2$$

we must have

$$e_s = \frac{S}{E_s}$$

$$= \frac{X/2}{E/2(1+\nu)}$$

$$= (1+\nu)e_x \tag{5}$$

where E_s = shear modulus

e_s = maximum engineering shear strain

Comparisons between the maximum stress and maximum strain criteria are shown in stress and strain spaces in Figure 8 and 9, respectively. The parameter is the Poisson's ratio. Note when Poisson's ratio is zero, the two criteria coincide. When Poisson's ratio is different from zero, the two failure criteria are significantly different. The two criteria predict the same mode of failure by shear , but different modes and numerical values for all other biaxial stresses and strains. Thus the criteria do not uniquely define modes of

4. Quadratic Criteria for Isotropic Materials

For isotropic materials, quadratic failure criteria are derivable from the maximum strain energy and maximum distortional energy. For plane stress states, they are expressed as:

for strain energy:

$$\sigma_x^2 - 2\nu\sigma_x\sigma_y + \sigma_y^2 + 2(1+\nu)\ \sigma_s^2 = X^2 \tag{6}$$

or for distortional energy:

$$\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\sigma_s^2 = X^2 \tag{7}$$

where X is the uniaxial tensile strength, as before. We can see that (7) is a special case of (6), when the Poisson's ratio is 1/2; i.e., for incompressible materials, distortional energy is the total strain energy. The maximum shear strength S can be related to the uniaxial strength X using (6):

$$2(1+\nu)S^2 = X^2, \text{ or}$$

$$\sqrt{2} \leq X/S \leq \sqrt{3}, \text{ or } .71 \leq S/X \leq .58 \qquad (8)$$

This relation is slightly different from that in (4), where the ratio of the strengths is 2 or .5, respectively. Graphically, (6) is shown in Figure 10 for Poisson's ratio equal to zero and 1/2. They are repeated in Figure 11 together with the maximum stress (Tresca) criterion. In this figure, the shear strength ratios of .5,

.58 and .71 cited in (4) and (8) are also shown. Similiar to Figure 5, the quadratic criterion has circular cross sections generated about the p-axis. Thus, the maximum stress criterion is not very different from the maximum distortional energy criterion.

The quadratic failure criterion in stress space can also be represented in strain space by introducing the usual stress-strain relation in plane stress into (6), the resulting maximum strain energy criterion is

$$\varepsilon_x^2 + 2\nu\varepsilon_x\varepsilon_y + \varepsilon_y^2 + 2(1-\nu)\ (\varepsilon_s/2)^2 = (1-\nu^2)\ e_x^2 \qquad (9)$$

The strain transformation is satisfied by this failure criterion for all values of Poisson's ratio. We can also show that

$$e_s/e_x = \sqrt{2(1+\nu)} \qquad (10)$$

Since Poisson's ratio varies from zero to one-half, we have an equation like (8)

$$\sqrt{2} \leq e_s/e_x \leq \sqrt{3} \qquad (11)$$

where e_s = maximum shear strain; e_x = maximum longitudinal strain under uniaxial stress. The failure criteria in strain space are plotted in Figure 12. The limiting Poisson's ratio of zero and one-half for isotropic materials are shown.

If tensile and compressive strengths for an isotropic material are different, the resulting maximum stress criterion is shown in Figure 13, where X' is the uniaxial compressive strength. The quadratic failure criterion can

also be modified to include linear terms of stress or strain components. The resulting failure envelope will be a displaced ellipsoid along the p-axis. The displaced center can be interpreted as the presence of initial stresses in the energy criterion. Shown in Figure 13 are 3 criteria fitting the same strength data. The predicted shear strengths based on the tensile-compressive strengths for the 3 criteria differ significantly (greater than those in Figure 11). Furthermore, shear strength varies along the p-axis, i.e., it is coupled with the absolute values of the tensile and compressive strengths. This is often called the Colomb failure criterion.

4. Maximum Stress and Maximum Strain Criteria for Orthotropic Materials

For orthotropic materials subjected to plane stress loading, it is normally assumed that at least five strengths must be measured; viz:

Longitudinal tensile strength, X

Longitudinal compressive strength, X'

Transverse tensile strength, Y (12)

Transverse compressive strength, Y'

Longitudinal/transverse shear strength, S

Having the measured strengths based on the ultimate values or some definable yielding, we can easily construct the failure criteria based on extensions of those for isotropic materials. The maximum stress criterion is shown in Figure 14. The corners in the second and fourth quadrants are cut off by dashed lines. One justification of cutting off the corners is that in the limit when orthotropy disappears we can recover the maximum stress criterion for the isotropic material shown as dashed lines in Figure 13. We cannot begin with a rectangle in Figure 14 to recover a hexagon in Figure 13, or vice versa. Stress transformation relation is not used here because shear strength for an orthotropic material is assumed to be an independent parameter, not related to the strength measured under a combined tensile-compressive stress.

Similarly, we can construct a maximum strain failure criterion using the same measured strength data in (12). Assuming linear stress-strain relation up to failure or yielding, we have the following failure strain parameters:

$$\begin{aligned}
&\text{Longitudinal tensile strain, } e_x = X/E_x \\
&\text{Longitudinal compressive strain, } e_x' = X'/E_x \\
&\text{Transverse tensile strain, } e_y = Y/E_y \\
&\text{Transverse compressive strain, } e_y' = Y'/E_y \\
&\text{Longitudinal/transverse shear strain, } e_s = S/E_s
\end{aligned} \tag{13}$$

With these strains, we can construct the maximum strain failure criterion in strain space. Note the uniaxial tensile and compressive tests follow loading lines with slopes equal to the longitudinal and transverse Poisson's ratios. We can cut off the corners shown by the dashed lines if we invoke the same rationale as we did in Figure 14. We can then claim that the maximum strain criterion for an orthotropic material in Figure 15. In the limit it becomes that for an isotropic material in Figure 7, when orthotropy disappears and tensile and compressive strengths are equal.

Traditionally, the maximum stress or maximum strain criterion for an orthotropic material is represented by a rectangular box having six plane faces. Since shear strength is symmetrical, there are only five independent plane faces. But if we invoke the hexagonal cross sections by cutting off the corners in Figure 14 or 15, we have an octahedral. With symmetry in shear, there will be seven independent plane faces. To use the maximum stress or maximum strain failure criterion, we need seven inequalities, which is numerically time consuming. The quadratic criterion is much easier to use because it is a single-valued fraction.

If we rotate the symmetry axes of our orthotropic material, we can define the resulting failure envelope by applying the stress or strain transformation

to the respective failure criterion. We can easily show that the failure envelope undergoes a rigid body rotation about the p-axis as the symmetry axes of the orthotropic material rotates. The p-axis is shown in Figures 14 and 15. This is exactly the same axis for the Mohr's circle rotation where the angular displacement is twice that of the reference or symmetry axes. This is shown in Figure 16 where the projected view of p = 0 plane is also shown.

The first-ply failure (FPF) envelope is the innermost locus of a multi-directional laminate consisting of plies with arbitrary orientations. Because of the rigid-body rotation about the p-axis, the FPF envelope approaches a circular cylinder generated about the p-axis as the number of ply orientations increase. For the maximum stress or strain criterion, the FPF envelope will also be a circular cylinder, the radius of which along the p-axis is the minimum value of R defined in (3). This minimum value may be one-half of the shear strength; i.e., the value along the r-axis (or q = 0) in Figure 16. The minimum value may also be the smallest q for a given p in the r = 0 plane.

5. Quadratic Criteria for Orthotropic Materials

The quadratic failure criteria for orthotropic materials has been proposed by many workers for at least 20 years. One often quoted source is that by Goldenb and Kopnov [1] and other workers in the USSR. It was postulated that the most general failure criterion in terms of stress components would be:

$$[F_i\sigma_i]^{\alpha} + [F_{ij}\sigma_i\sigma_j]^{\beta} + [F_{ijk}\sigma_i\sigma_j\sigma_k]^{\gamma} + \ldots = 1 \tag{14}$$

The quadratic criterion, in a simplified form, is

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1 \tag{15}$$

It is easier to use strain space to represent the failure envelopes of multi-directional composite laminates because laminate strains and ply strains are equal under in-plane stresses. Thus, strain space envelopes are independent of the ply orientations in a laminate; e.g., a 0-degree envelope is fixed in strain space whether or not other ply orientations exist in a given laminate. The conversion from stress to strain space is simple if the stress-strain relation to failure is linear. This is a good approximation for most composite materials. We can show the quadratic criterion of (15) in strain ace [2] as:

$$G_i \varepsilon_i + G_{ij} \varepsilon_i \varepsilon_j = 1 \tag{16}$$

where

$$G_i = Q_{ij} F_j$$

$$G_{ij} = Q_{ik} Q_{j\ell} F_{k\ell}$$

Q_{ij} = plane stress stiffness modulus

By using uniaxial and pure shear tests, the nonzero components of F_{ij} and F_i can be calculated from the measured strengths in (12), as follows:

$$F_{xx} = 1/XX' \ , \ F_{yy} = 1/YY' \ , \ F_{ss} = 1/S^2$$
$$F_x = 1/X - 1/X' \ , \ F_y = 1/Y - 1/Y' \tag{17}$$

Every material parameter in (15) can be determined by simple tests except the interaction term F_{xy}. In the absence of a truly biaxial test, we can only postulate its value. It is easier to assess the value of this interaction term by its normalized value. We define:

$$F^*_{xy} = \text{normalized interaction term}$$

$$= F_{xy} / \sqrt{F_{xx} F_{yy}} \quad (18)$$

The quadratic failure envelope is an ellipsoid and is constrained geometrically by

$$-1 < F^*_{xy} < 1 \quad (19)$$

The quadratic envelope becomes a hyperboloid when the normalized interaction term becomes greater than 1 or less than -1. At the absolute value of unity, we have two parallel planes.

Hill [3] proposed a special form of (15)

$$F_{ij}\sigma_i\sigma_j = 1 \quad (20)$$

This was intended for orthotropic materials with equal tensile and compressive strengths. The linear terms in (15) are zero for this case. There is another assumption in Hill's postulate that

$$F_{xy} = -1/2X^2, \text{ or } F^*_{xy} = -Y/2X \quad (21)$$

For highly orthotropic materials; i.e.

$$X >> Y, \text{ or } F^*_{xy} \simeq 0 \quad (22)$$

Hoffman [4] kept the linear terms in (15) but used the same near zero interaction term as in (21)

We proposed [5] a more general interaction term than that in (21). A generalized von-Mises criterion for orthotropic materials was defined by [2]:

$$F^*_{xy} = -\frac{1}{2} \quad (23)$$

We can also show graphically that the quadratic envelopes for most composite materials are "well behaved" if the normalized interaction term is bounded by

$$-1/2 \leq F^*_{xy} \leq 0 \tag{24}$$

These limits are more restrictive than the maximum bounds in (19). The limits of (24) would ensure that the envelopes are not excessively elongated, and, in addition, are oriented in the first and third quadrants similar to the tilt of isotropic materials in Fig. 10, [6]. These bounds are also consistent with those for Poisson's ratios of isotropic materials. If we wish to recover the failure criterion based on strain energy in the limit, the bounds in (24) would be appropriate. It is implied here that the normalized interaction term may be related to the effective Poisson's ratios of the orthotropic ply, or that of a quasi-istropic laminate consisting of the orthotropic plies.

Using the strength data for a typical graphite-epoxy composite T300/5208 from [2], the quadratic failure envelopes for a laminate with 4 ply orientations: 0, 90, ±45 degrees are plotted in Figures 17 and 18 with values of F^*_{xy} equal to -1/2 and 0, respectively. Although the outlines of the envelopes are different, the FPF (first-ply failure) envelopes remain insensitive to F^*_{xy} within the range specified by (24). Since buckling may intercede, the third quadrant (compression-compression straining) is excluded from this comparison. Figure 17 the generalized von Mises [2]; Figure 18, essentially Hoffman's [3].

The maximum stress and maximum strain criteria for this composite material (T300/5208) can also be plotted using the same data as those in Figure 17 and 18. All six failure criteria are plotted in Figure 19. Again, the FPF envelopes are not significantly different from the quadratic criterion if the compression-compression quadrant is excluded. For the FPF envelope, the cross section at any constant p value approaches circular as the number of ply orientations increase. This envelope is approximately ellipsoidal for every criterion. We also see that the truncated maximum stress and maximum strain criteria, shown in Figs.14 and 15, yield nearly the same FPF envelope as the "box " criteria. Thus, if we deal with multidirectional laminates as we normally do, the resulting FPF

envelope is insensitive to the failure criteria which we have discussed. This leads us to select the quadratic criteria as the preferred based primarily on two points:

- They are easy to use by virtue of being single valued functions.
- They are scalar products of tensors, which are mathematically sound and have all the appropriate invariants and transformation properties established and ready to be used without further proof.

The uncertainty of the interaction term is not a severe limitation because for most materials the effect on the FPF envelope is negligible.

6. Experimental Data

Most data in composite laminates are derived from uniaxial tension tests. Off-axis unidirectional data exist extensively. It is well known that the predicted off-axis strengths by the maximum stress, maximum strain and quadratic criteria, and the measured data are all in close agreement to one another. Thus the off-axis data cannot be used to determine which criterion is best.

The next series of uniaxial tests are usually the biaxial laminates; viz., cross-ply (0 and 90 degrees) or angle ply ($\pm\phi$) laminates. The ply stress is now quite different from the uniaxial laminate stress. Each ply is subjected to biaxial or combined state of stress. More different ply stresses can be induced by using tri-directional laminates such as 0 or 90, and $\pm\phi$ degrees. Soni [7] made comparisons of the uniaxial tensile strength of tri-directional laminates. The results are shown in Figure 20 and 21. The FPF predictions by various crite remain approximately the same. Measured strength data in Figure 20 were higher than the FPF value. The 0-degree ply in the laminate continued to carry load after the FPF. The measured strengths in Figure 21 were essentially those FPF predictions. The laminate failed to carry more load after the FPF. The "unbroken" angle plies, unlike 0-degree plies in Figure 20, could not carry

the prevailing load after the FPF.

We believe that failure criteria are not valid after the FPF. Plies within a laminate become broken by internal damage such as transverse cracks and delaminations which accumulate as applied load increases. The classical laminated plate theory is not strictly valid for broken, discontinuous materials. By reducing ply stiffness we can simulate degraded, damaged plies. This is only approximate, and cannot be used to describe failure. A more accurate theory that reflects a damaged material is needed. In the absence of a realistic theory, we recommend systematic testing to ensure safety of structures. While Figure 20 shows that the degraded stiffness approach after the FPF may be valid, Figure 21 completely refutes this approach; post-FPF strength is zer

Kim [8] showed that if edge delamination of a multi-directional laminate coupon is prevented by edge reinforcement, the measured strength is increased considerably and approaches that predicted by the quadratic failure criterion. Delamination, as a form of internal damage, is detrimental to the laminate strength. This mode of failure, however, can be easily eliminated by edge reinforcement.

Based on available data, it can be concluded that

- Small difference exists among various failure criteria. This is particularly true for the FPF envelope which is approximately ellipsoidal for all criteria.
- Predicted uniaxial strengths of multi-directional laminates can be confirmed by experimental data if internal damage is kept to a minimum. The damage will lower the laminate strength.
- We believe that the quadratic criterion is the easiest to use because it is a single-valued function. The maximum stress and maximum strain criteria are numerically difficult to use whether it is a cube or

octahedron. Since all criteria yield about the same answer and are not related to the mechanisms of failure, why not use the easiest?

- Although all failure criteria are empirical in nature, the quadratic criteria are the best mathematically from the standpoint of invariance, symmetry, algebraic forms, and geometric shapes.

7. Other Criteria

There are many practical considerations to failure criteria. They must be easy to use. They must contain minimum number of material constants. They must provide smooth transition from orthotropy to isotropy, or vice versa. The truncation of the "boxy" criteria is done to ensure this smooth transition. Forced empirical fit from one set of data in one quadrant may lead to a bad fit in another quadrant.

In the absence of biaxial tests, the interaction term F_{xy} is a floating constant. Chamis [9] proposed the use of different F_{xy} for different quadrants. With additional constants, the failure envelope is customized for each quadrant. Using the same strength data of T300/5208 from [2], the customized failure envelope in strain space is shown in Figure 22, for a 0, 90, ±45 degrees laminate. The failure envelope is segmented and not well behaved. The FPF envelope however is not significantly different from those of other criteria in Figure 19.

Rosen [10] also suggested the use of different F_{xy} for different quadrants. Beyond having more coefficients for better data fit, there is no physical or mathematical justification. In fact, mathematically the functional dependency of the stress components such as that in (14) and (15) is committed. The signs of the stress components are built in. To the

first order approximation, all strength parameters F_{ij} and F_i are constants and cannot vary wi

sign of the stress components. A higher order approximation may bring in the dependency of the strength parameters on the stress invariants. But such higher order theory brings with it many other complications beyond the utility of failure criteria. We cannot see justification for tampering with F_{xy}.

If more constants are needed for better data fit, there are two more straight forward yet mathematically consistent options than the use of multiple interaction terms suggested by Chamis [9] and Rosen [10]. The easy options are the three-dimensional quadratic criterion for a transversely isotropic material, and the two-dimensional cubic criterion.

We can easily extend from the plane stress criterion to the 3-dimensional criterion for a transversely isotropic material. Assuming that the y-z plane is isotropic, (15) can be written as:

$$F_{xx}\sigma_x^2 + F_{yy}(\sigma_y^2 + \sigma_z^2) + F_{ss}(\sigma_s^2 + \sigma_u^2) + F_{tt}\sigma_t^2$$

$$+ 2F_{xy}(\sigma_y + \sigma_z)\sigma_x + 2F_{yz}\sigma_y\sigma_z$$

$$+ F_x\sigma_x + F_y(\sigma_y + \sigma_z) = 1 \qquad (25)$$

where $\sigma_u = \sigma_{xz}$, $\sigma_t = \sigma_{xz}$. Similar to (17) we can show that

$$F_{tt} = 1/T^2$$

$$F_{yz} = F_{yy} - 1/2\, F_{tt} = \frac{1}{YY'} - \frac{1}{2T^2} \qquad (26)$$

where T is the shear strength in the y-z plane. For a quadratic 3-dimensional criterion of a transversly isotropic material we only need one more constant T, the transverse -transverse shear strength. But the more difficult problem will be the 3-dimensional stress analysis to determine σ_u (σ_{xt}) , σ_t (σ_{yz}) and σ_z.

Another approach to a higher order failure envelope is to use the "cubic" terms in (14). Here sixth order strength parameters are needed. Wu [6] and Tennyson et al [11] showed that with four additional constants

$$F_{xxy},\ F_{yyx},\ F_{xss},\ F_{yss} \qquad (27)$$

the cubic criteria in (14) can provide improved fit of data. But the measurement of four additional constants is not easy. Although the need for the cubic terms is not settled, their impact on the FPF is not significant.

8. Optimized Plies

An optimized ply from the strength standpoint is achievable if the failure criterion of the ply is invariant; i.e., it is the same for all ply

orientations. All plies would fail simultaneously; i.e., the FPF, the LPF (last ply failure), and the IPF (intermediate ply failure) will be coincident.

How do we do it? Do we improve the laminate strength after we produce this optimized ply?

Knight [12] used the quadratic failure criterion of (15) and derived the exact relations for optimized plies. The simple key is to produce a failure criterion of a 0-degree ply that is symmetrical about the p-axis in strain space. Since ply rotation by θ is a rigid body rotation about the p-axis by 2θ, failure criteria for all ply orientations will be coincident. The shear strength must have such a value that would ensure circular crosssections for any value along the p-axis.

The failure envelopes of a glass-epoxy composite Scotchply 1002, with ply orientations between 0 and 90 degrees at 15-degree intervals are shown in Figure 23. The strength data for this composite is taken from [2]. The basic tensile and compressive failure strains defined in (13) are shown as black dots. As we go from a, b, c to d, we progressively optimize the failure strains. The disparity of failure envelopes is reduced. Ultimately, a fully optimized ply has only one failure envelope independent of the ply orientation. Using the quadratic criterion, we can collapse failure envelopes of different ply orientations onto one envelope. This is not possible if the maximum stress or maximum strain criterion is used, because the rotation of a cube or octahedron does not create a smooth envelope.

Finally, we need to know if an optimized ply is in fact stronger than the original unoptimized ply. This answer is a definite yes. The increase in the FPF envelope in Figure 23 is dramatic. We can also show the increase in stress space, although the strength improvement is smaller. The increase in the ultimate strains does not take into account the possible reduction in stiffness. The net result is that the effective strength in stress space is increased by a lesser degree. In terms of materials engineering, an optimized ply usually calls for a dramatic increase in the transverse tensile strain in most current composite materials. A softer matrix is in the proper direction to improve laminate strength. Woven fabric is a good composite material because the failure strains are nearly identical in two principal directions.

8. Summary

We would like to summarize our views on macroscopic failure criteria for orthotropic materials.

- All criteria provide reasonable and nearly identical prediction of the FPF (first-ply failure) ; e.g., see Figure 19.
- Prediction of failure subsequent to FPF is not always reasonable or reliable ; e.g., see Figure 21.
- Extensions of the quadratic criterion beyond the present can be done. Simple examples include the 3-dimensional quadratic criterion or the 2-dimensional cubic criterion.
- Ply properties can be optimized by creating symmetry about the p-axis. This trend is independent of the choice of failure criteria.

We recommend the quadratic criterion for the following reasons:

- It is easy to use.
- It is a single-valued function. It is particularly suited for numerical solution, such as the use of strength ratios [2].

- It is based on mathematically rigorous framework; i.e., scalar products, coordinate transformation, invariants, and symmetry are well established entities and easy to work with. Extension to 3-dimensional criteria for a transversely isotropic material in (25) is simple.

It is not possible to relate the complex mechanisms and modes of failure to all the macroscopic failure criteria mentioned in this survey. It is safe to say that interactions exist among the various mechanisms and modes. Non-interactive criteria such as the maximum stress and maximum strain are undoubtedly gross simplifications. The quadratic criteria have some interactions built in, but are far from adequate to deal with the exact mechanics of failure. But quadratic criteria are the best we have and they serve the purpose of design, guidelines for materials and processing improvements.

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FIGURE CAPTIONS/TSAI

Fig. 1. Uniaxial Tension Test for a Linear Material

Fig. 2. Maximum Stress Criterion with Equal Tensile and Compressive Strengths

Fig. 3. Maximum Stress Criterion with Compressive Strength Higher than Tensile Strength. The Stress-Strain Curve is Shown on the Left.

Fig. 4. Mohr's Circle in Stress Space Showing the Relation Between Tension-Compression and Pure Shear States of Stress ; σ_s is Shear Stress

Fig. 5. Maximum Stress Criterion for Isotropic Materials. The Equivalence of the Tension-Compression and Shear Strengths is Shown. The Failure Surface in the q-r Space must have Circular Crosssections to Satisfy the Mohr's Circle Relations.

Fig. 6. Biaxial or Combined Failure Strains of an Isotropic Material Resulting from Uniaxial Tensile or Compressive Stress.

Fig. 7. Maximum Strain Criterion for an Isotropic Material with Equal Tensile and Compressive Strengths. The "Box" Criterion is Modified to Satisfy Strain Transformation Relation Similar to the Modified Maximum Stress Criterion in Fig. 5.

Fig. 8. Maximum Strain Criteria for Different Poisson's Ratios in Stress Space. The Slopes and Coordinates Intercept for the Case of ν=1/3 are Shown.

Fig. 9. Using the Same Combined Failure Strains, Maximum Stress Criteria for the Limiting Poisson's Ratios of an Isotropic Material in Strain Space.

Fig. 10. Quadratic or Energy Criteria for an Isotropic Material in Stress Space with the Limiting Poisson's Ratio.

Fig. 11. The Relation Between the Tension-Compression and Shear Strengths are Different Depending on the Failure Criteria and the Poisson's Ratio.

Fig. 12. Limiting Quadratic Failure Criteria in Strain Space. The Tilt of the Ellipse is Different from that in Fig. 10 for the Incompressible Material Because the Sign of the Interaction Shown in (6) and (9) are Different.

Fig. 13. Failure Criteria for an Isotropic Material with Different Tensile and Compressive Strengths. The Maximum Stress Criterion is in Dashed Lines; the Quadratic Criteria can be Fitted with Different Displacement of the Center and Different "Effective Poisson's Ratios ν."

Fig. 14. Maximum Stress Criterion for an Orthotropic Material with Different Tensi and Compressive Strengths. The "Box" Criterion is Shown in Solid Lines; the Truncated Criterion Shown in Dashed Lines.

Fig. 15. Maximum Strain Criterion for an Orthotropic Material. The Corners in the Second and Fourth Quadrants have Been Truncated by Dashed Lines.

Fig. 16. A Rigid Body Rotation of Failure Envelopes About the p-axis (from q to q' axis) by 2θ as the Material Symmetry Axis Rotation (from X to X' axis) of an Orthotropic Material by θ.

Fig. 17. Quadratic Failure Criterion of 0, ±45, and 90 Degree Ply Orientations of T300/5208 Using $F^*_{xy} = -1/2$.

Fig. 18. Quadratic Failure Criterion of 0, ±45, and 90 Degree Ply Orientations of T300/5208 Using $F^*_{xy} = 0$.

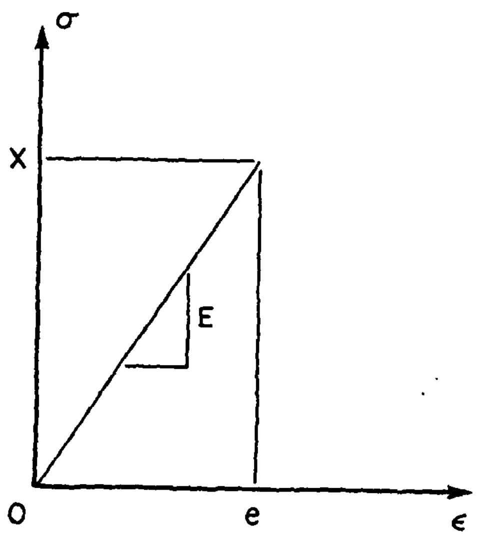
Fig. 19. Comparison of Failure Criteria for T300/5208 with 0, ±45, and 90 Ply Orientations. Both the "boxy" and truncated Maximum Stress and Strain Criteria are Shown. The predicted FPF is shown in shaded area.

Fig. 20. Prediction and Data of AS/3501 Tri-Directional Laminats [0/±Ø] from [7].

Fig. 21. Prediction and Data of AS/3501 Tri-Directional Laminats [90/±Ø] from [7].

Fig. 22. The Segmented Quadratic Failure Criteria with Different Interaction Terms in Different Quadrants [9].

Fig. 23. Gradual Improvement in Strength of Scotch-Ply 1002 from (a), (b), (c), to (d), the Fully Optimized Ply where Failure Envelope of all Ply Orientations Coincide.



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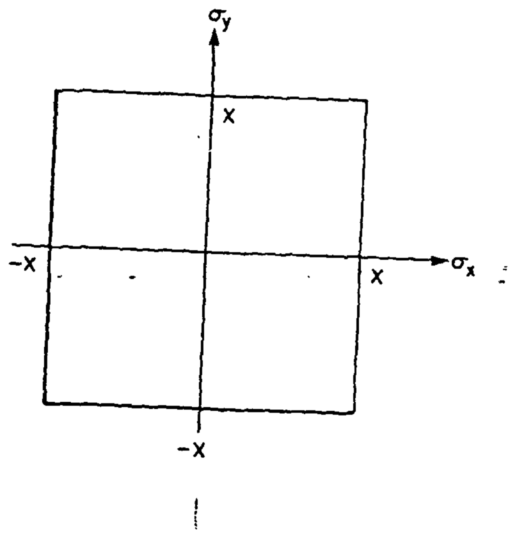

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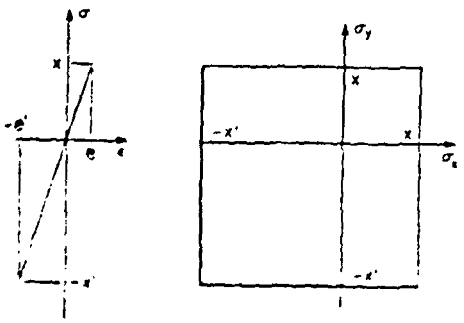

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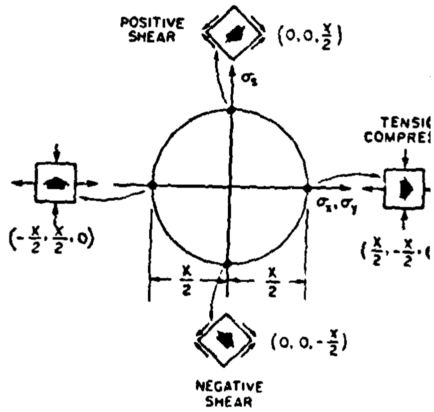

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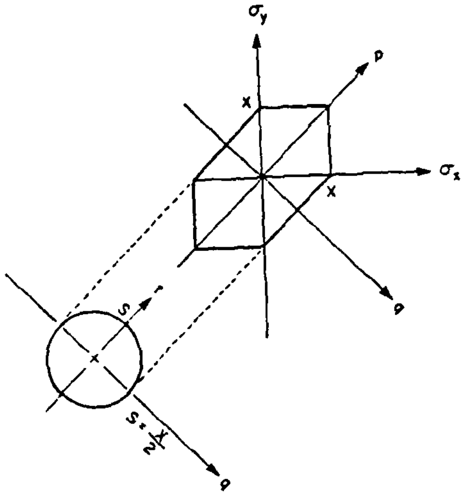

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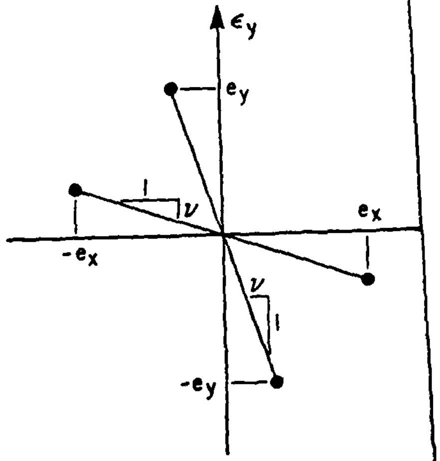

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Fig 6

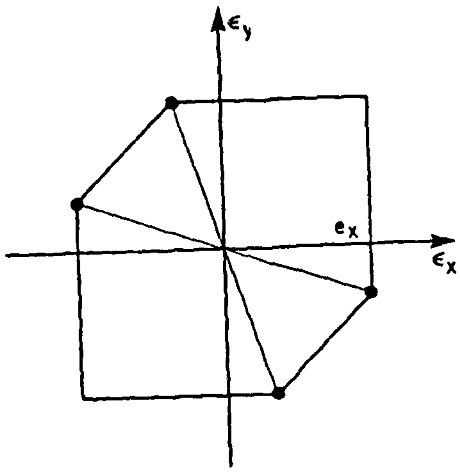

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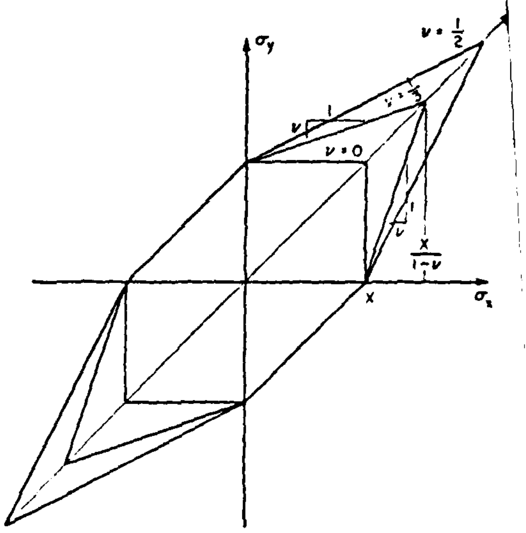
Fig. 7



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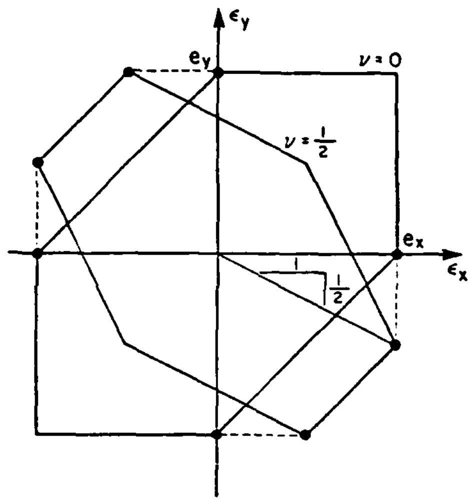

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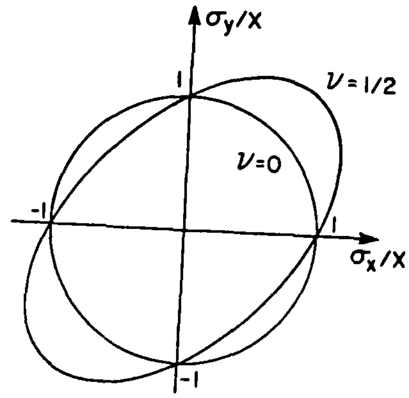


Fig. 10

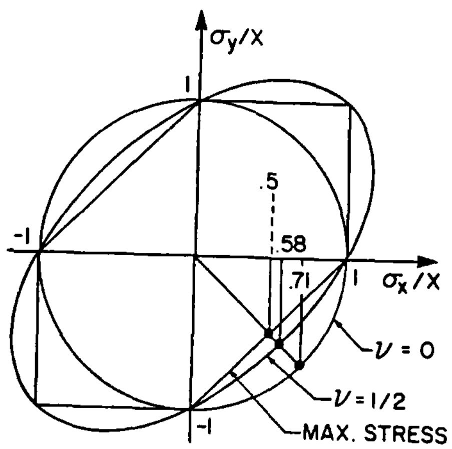
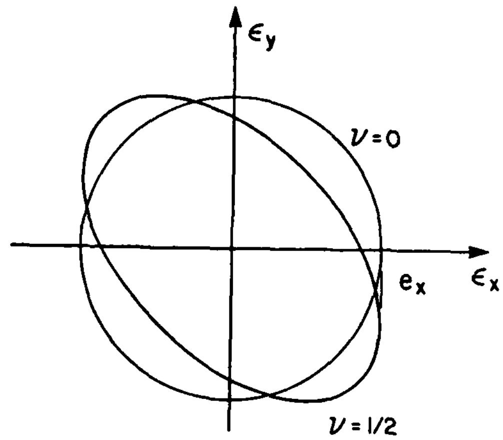


Fig. 11



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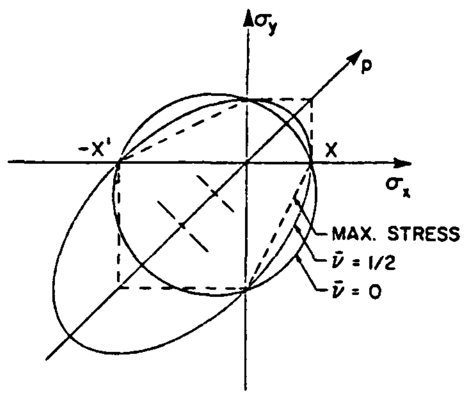

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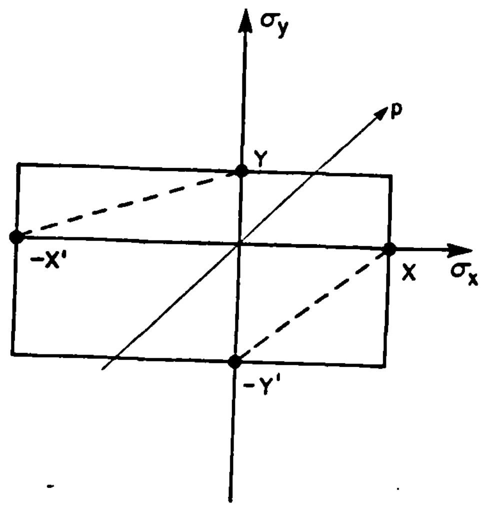
Fig. 13



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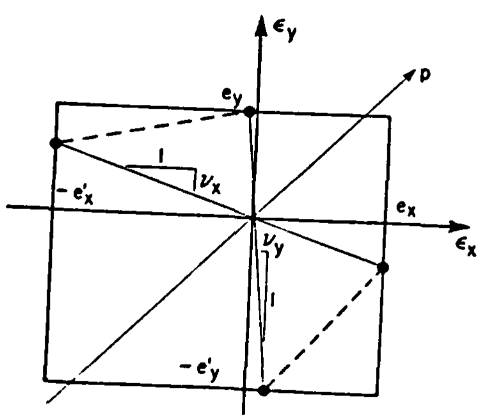

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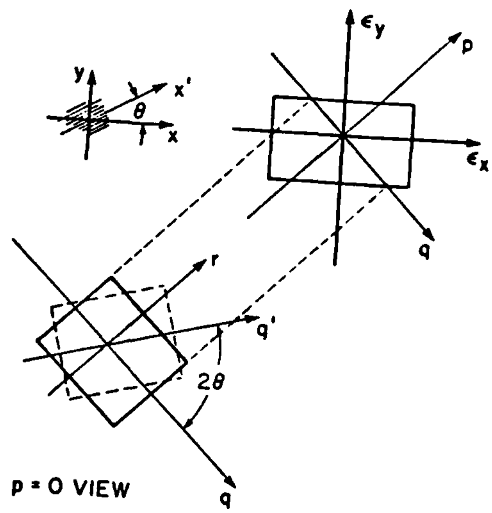

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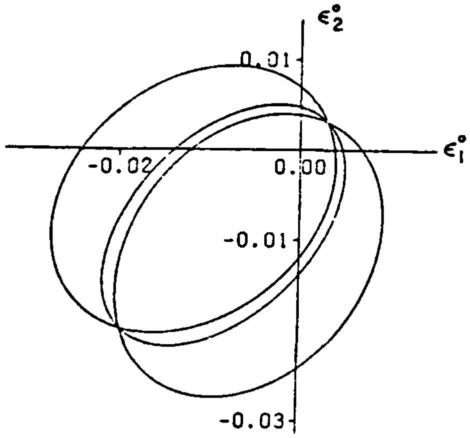

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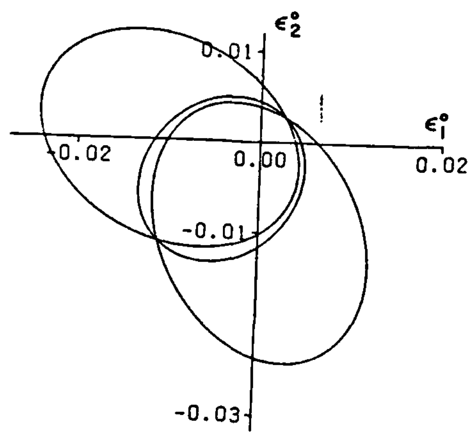

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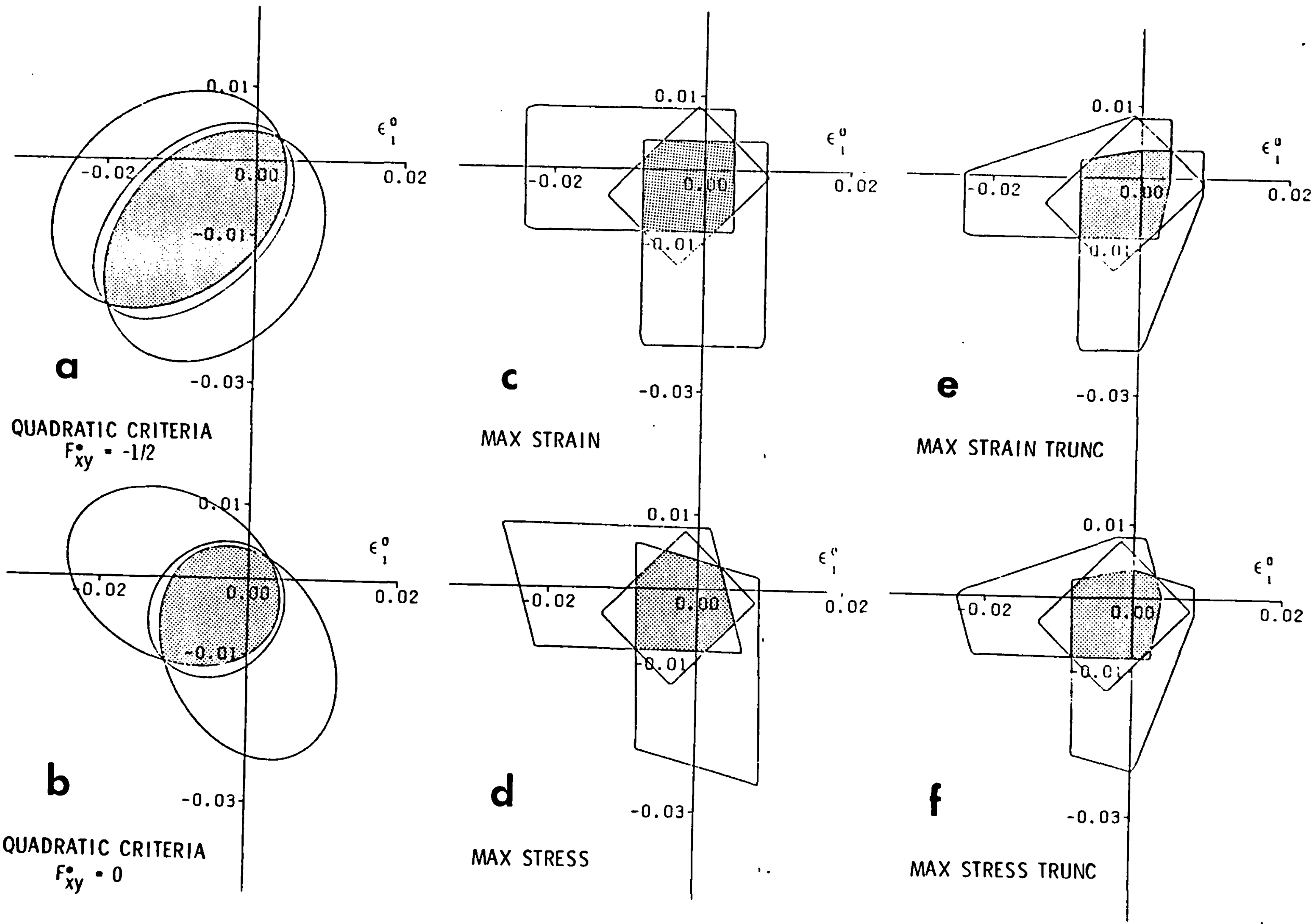
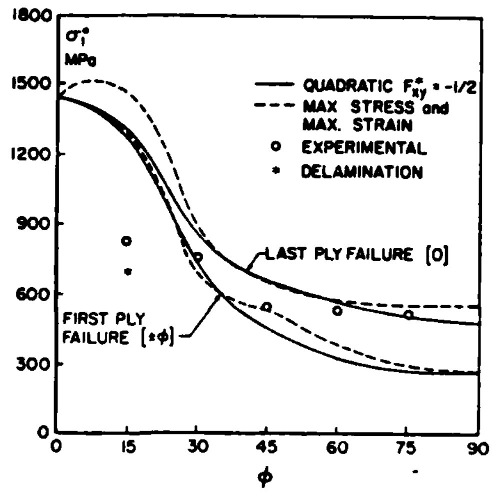

a
QUADRATIC CRITERIA
$F^{*}_{xy} = -1/2$
b
QUADRATIC CRITERIA
$F^{*}_{xy} = 0$
c
MAX STRAIN
d
MAX STRESS
e
MAX STRAIN TRUNC
f
MAX STRESS TRUNC
ϵ_1^0
0.01
0.00
-0.01
-0.02
-0.03
0.02



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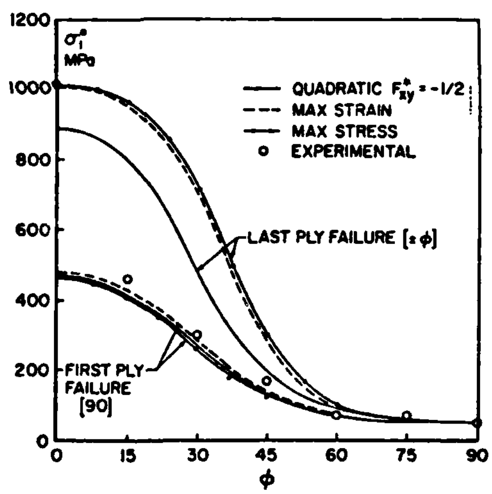

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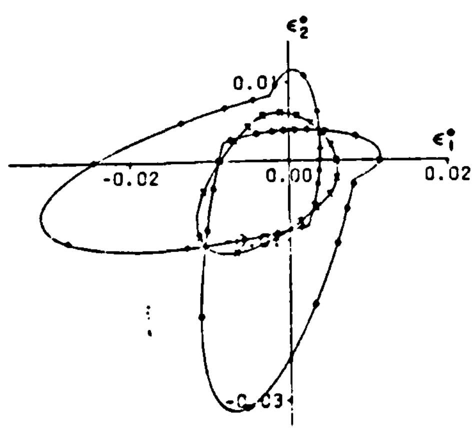

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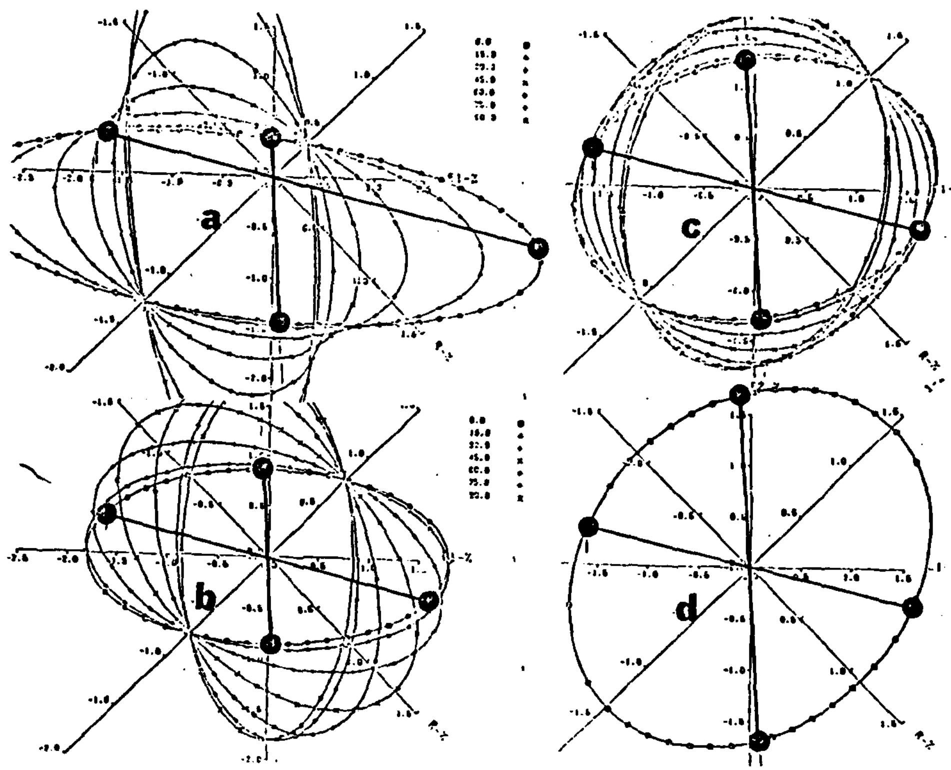

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